Gravitational Duals from Equations of State

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Abstract

Holography relates gravitational theories in five dimensions to four-dimensional quantum field theories in flat space. Under this map, the equation of state of the field theory is encoded in the black hole solutions of the gravitational theory. Solving the five-dimensional Einstein field equations (EFE) to determine the equation of state is an algorithmic, direct problem. Determining the gravitational theory that gives rise to a prescribed equation of state is a much more challenging, inverse problem [1]. We present a novel approach to solve this problem based on physics-informed neural networks (PINNs). The resulting algorithm is not only data-driven but also informed by the physics of the Einstein's equations. We successfully apply it to theories with crossovers, first- and second-order phase transitions.

Theoretical model

We consider a 4-D Conformal Field Theory (CFT) deformed by a relevant operator $\mathcal{O}$. The holographic dual consists of 5-D gravity coupled to a scalar field ϕ with a potential $V(\phi)$ [3]. The bulk action:

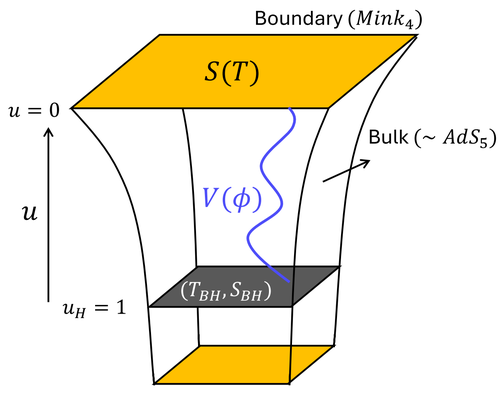
$$S = \frac{2}{\kappa^2}\int dx^5\sqrt{-g}\left[\frac{1}{4}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] . \qquad (1)$$

For specific $V(\phi)$, the thermodynamics of the gauge theory is determined by finding all the static, planar black hole solutions of (1). We consider an asymptotically anti-de-Sitter (AdS) spacetime bulk metric

$$ds^2 = -A(u)\,dt^2 + \Sigma^2(u)(dx^2+dy^2+dz^2) - 2u^{-2}\,dt\,du\ , \qquad (2)$$

where u is the null holographic coordinate. The temperature and the entropy density of the gauge theory can be computed from the surface gravity and the area density of a black hole solution in the bulk:

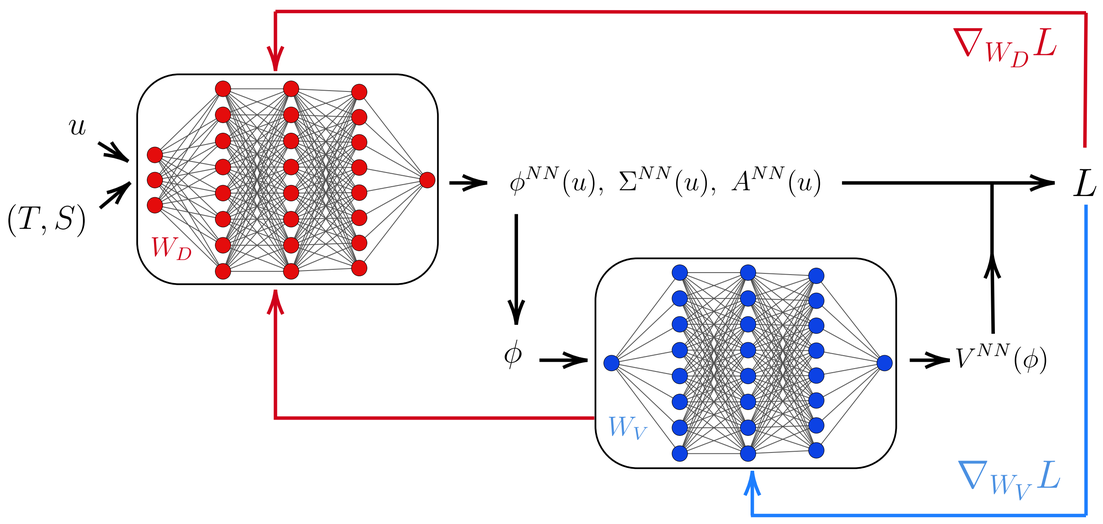
$$T = T_{\rm BH} = \frac{-1}{4\pi}A'(u=1)$$
$$S = S_{\rm BH} = \frac{2\pi}{\kappa^2}\left[\Sigma(u=1)\right]^3$$



Given (S,T), we set different BCs for the Einstein Field Equations (EFE). The aim is to obtain the unique $V(\phi)$ such that the EFE solutions satisfy all BCs.

Methods

A neural network (NN) acts as a function, taking an input and performing a non-linear transformation depending on tunable parameters W. In PINNs each unknown function will be a NN [2]:



The loss function consists on the squared residuals of EFE. We sum over all sampled points in the independent variable u, and over all values of the BCs given by (S,T) points:

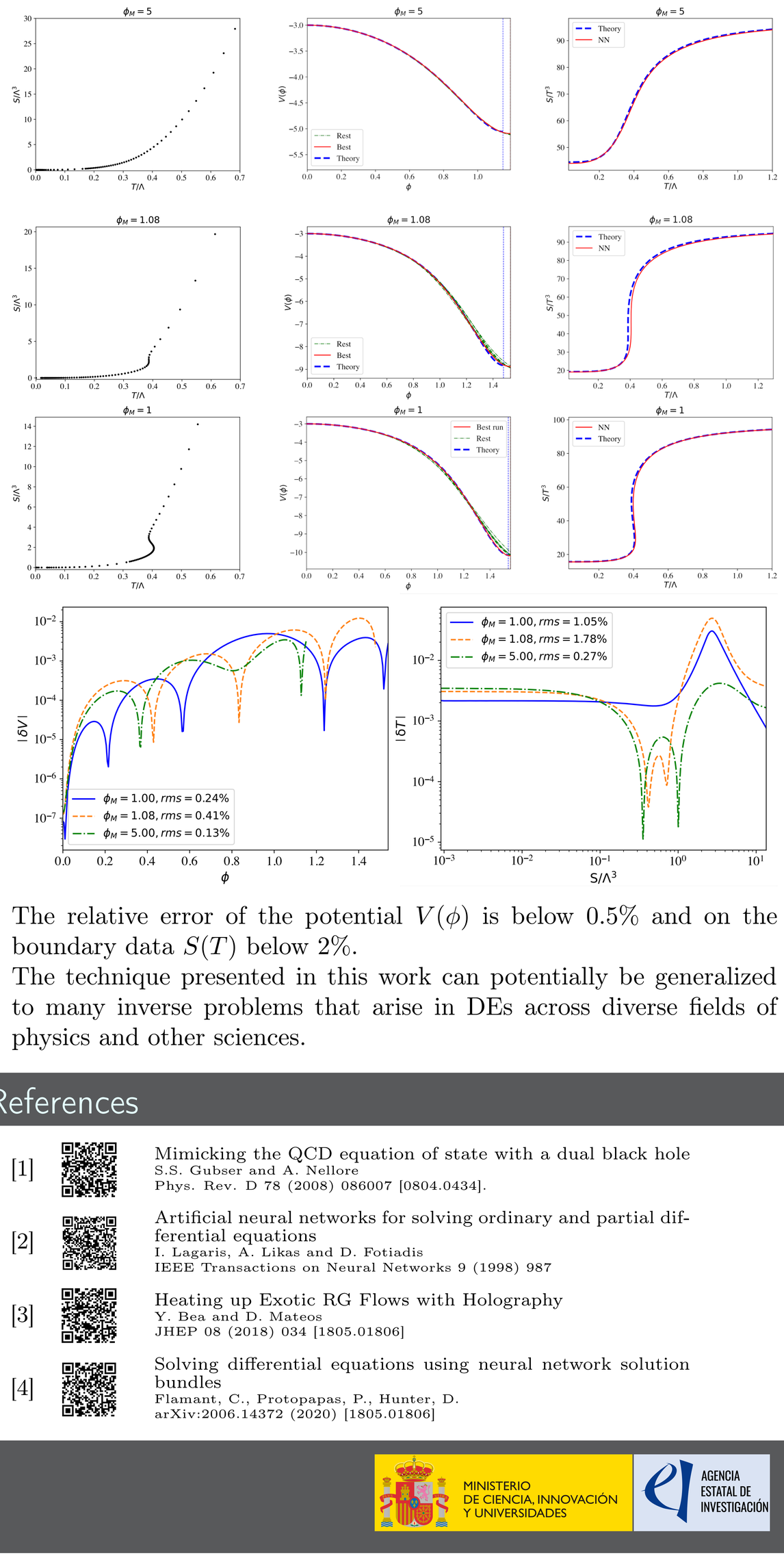
$$L = \sum_u \sum_{(S,T)} \left[EFE\left\{\phi^{\rm NN}, \Sigma^{\rm NN}, A^{\rm NN}, V^{\rm NN}(\phi)\right\}\right]^2 \qquad (3)$$

The loss is optimized via stochastic gradient descent. In this way, we learn the functional form of solutions for different BCs all at once, in what is called a bundle solution [4]. This allows evaluation of the solutions for any (T,S) point independent of the points used during training.

Results

We test our setup with a crossover, a second- and a first-order phase transition. For each of them we recover a guessed potential and use it to re-uncover the thermodynamics of the gauge theory. We quantify the error by comparing the potential with the theoretical one, and the original boundary data with the recovered one.

$$S(T)_{\rm input} \quad \rightarrow \quad V_{\rm NN}(\phi) \quad \rightarrow \quad S_{\rm NN}(T)/T^3$$



The relative error of the potential $V(\phi)$ is below 0.5% and on the boundary data $S(T)$ below 2%.
The technique presented in this work can potentially be generalized to many inverse problems that arise in DEs across diverse fields of physics and other sciences.

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